

Econ 82100 Econometrics I
Homework 6: Last Year's Final
(due December 5)

1. Suppose the regression model is $y_i = \mu + \epsilon_i$, where $E(\epsilon_i|x_i) = 0$, $Cov[\epsilon_i, \epsilon_j|x_i, x_j] = 0$ for $i \neq j$, but $Var[\epsilon_i|x_i] = \sigma^2 x_i^2$, $x_i > 0$.

a) (10 points) Given a sample of observations on y_i and x_i , what is the most efficient estimator of μ , What is its variance?

b) (10 points) What is the OLS estimator of μ , and what is the variance of the ordinary least squares estimator?

c) (10 points) Prove that the estimator in part a is at least as efficient as the estimator in part b.

d) (5 points) Given that the error variance possibly depends on x_i^2 , how would you test for the existence of heteroscedasticity? illustrate one method.

2. Consider the random variable x_t that has a density function $f(x_t) = \lambda x_t^{-(\lambda+1)}$, $1 \leq x_t \leq \infty$, $\lambda > 2$.

a) (10 points) Find the MLE of λ

b) (5 points) Find the variance of the MLE of λ

c) (10 points) Find the GMM estimator of λ using the first two moments (around origin) of x_t .

3. In the simultaneous equation models of the following:

$$\text{Demand equation: } q_i = \alpha_0 + \alpha_1 p_i + u_i$$

$$\text{Supply equation: } q_i = \beta_0 + \beta_1 p_i + \beta_2 x_i + v_i$$

where $E(u_i) = 0$, $E(v_i) = 0$, $cov(u_i, v_i) = 0$, q_i is quantity demanded or supplied, p_i is the price, and x_i is prices of raw material, $cov(x_i, v_i) = 0$, $cov(x_i, u_i) = 0$

a) (5 points) Solve for p_i and q_i from the two equations.

b) (5 points) Is the demand equation identified? Is the supply equation identified? Are they overidentified?

c) (10 points) Derive the IV estimator for α_1 or β_1 if they are identified.

4. (True or False and explain) (10 points) The OLS is a consistent estimator if the error follows an AR(1) process.

5. (True or False and explain) (10 points) 2SLS is an efficient GMM estimator.